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# CONTROL AND MANAGEMENT OF THERMO-MECHANICAL PHENOMENA DURING THE MECHANICAL PROCESSING OF PRODUCTS MADE OF MATERIALS WITH A NON-HOMOGENEOUS STRUCTURE

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*This article provides information technology on the analysis and synthesis of models for ensuring the quality characteristics of the working surfaces of products during mechanical processing, taking into account the defects of the material structure. Various micro defects are described, the development of which, under the influence of mechanical processing, leads to the appearance of cracks and their increase and, as a result, local or complete destruction. The problem of thermoelasticity for bodies weakened by inhomogeneities is solved. The probabilistic-statistical approach in increasing the accuracy of identification of technological processes of mechanical processing, development, and implementation of new, more effective methods and means of information and computer modeling is considered. The developed model takes into account the influence of inhomogeneities of technological origin (starting from the workpiece and ending with the finished product), which arise in the surface layer during the manufacture of structural elements, the analysis of which allows the creation of an information base of criteria, the implementation of which allows preventing the loss of functional properties of the responsible elements. Mathematical models of the dynamics of thermomechanical processes accompanying the mechanical processing of products from materials of the heterogeneous structure have been improved in the form of a spatially non-stationary formulation of the problem based on systems of differential equations of thermoelasticity in partial derivatives and discontinuity conditions on defects of the elastic-deformation characteristics of the processed material, which, unlike existing ones, made it possible to increase the accuracy of identification of mathematical models generally. Mathematical models of the system for evaluating the effectiveness of the functioning of technological complexes of mechanical processing have been developed, which make it possible to determine the relationship between the state parameters of the treated surfaces and the main controlling technological characteristics that provide the necessary properties of functionally gradient materials. Finally, the modeling results provide an opportunity to create effective information technology, which makes it possible to reduce the loss of functional properties of heterogeneous systems significantly.*

**Keywords:** structural element, surface layer, heat flux, processed material, controlled value.

## Introduction

Among the methods of researching physical processes occurring in environments of heterogeneous structure, electromagnetic signals in environments with variable characteristics, as well as the formation of defects on the working surfaces of products made of structural materials with various types of heterogeneity of hereditary origin, the development of information technologies for analysis and synthesis remains the most effective models for ensuring the quality characteristics of the working surfaces of products during mechanical processing.

**The work aims** to create an information technology that considers the control and management of thermomechanical phenomena during the mechanical processing of products from materials of the heterogeneous structure.

This work presents the information technology of functional connections of technological parameters with thermomechanical processes accompanying the processing of materials based on the analysis and synthesis of models for ensuring the quality characteristics of the working surfaces of products during mechanical processing. The only approach to solving problems of thermoelasticity for bodies weakened by inhomogeneities is presented thanks to the method of singular integral equations [1, 2].

The choice of a method for studying the strength and destruction of structural elements depends on the size of the object being studied, the limited state of fundamental elements weakened by defects, and the construction of information technology on this basis, which will create the prerequisites for a good display of the destruction mechanism.

Micro-research in manufacturing elements of workpiece structures related to inhomogeneities, which are formed in the surface layer at the production stage. Taking defectiveness into account allows us to adequately consider the mechanism of destruction of objects as a process of crack development. Therefore, in addition to the deterministic approach during the research, it is also necessary to consider a probabilistic statistical analysis of the effect of material inhomogeneities of structures on their functional properties [3, 4].

## Formulation of the problem

Let us consider some provisions necessary for modeling systems in a non-homogeneous space. The inner and outer regions separated by a system of closed contours included in the contour L are denoted by $S^+$ and $S^-$, respectively, while we consider that the outer region $S^-$ contains an infinitely distant point of the complex plane $C$. Let the function $f(t) \in H^{(\mu)}(L)$, where $H^{(\mu)}(L)$ is the set of Hölder continuous functions on the contour, that is, such that meet the conditions [5-7]:

$$|f(t_2) - f(t_1)| \leq A|t_2 - t_1|^{\mu}; \quad A > 0;\ 0 < \mu < 1. \tag{1}$$

The constant $A$ here is called the Hölder constant, and $\mu$ is the Hölder exponent. At the same time, at the points $z \in L$ of the complex plane $C$, there is a Cauchy-type integral as the primary value:

$$F(z) = \frac{1}{2\pi i}\int_L \frac{f(\tau)d\tau}{\tau-2},\ z \notin L, \tag{2}$$

moreover, at the points $z \in L$, this integral exists in the usual sense, and on the contour $L$, the relation [8] is valid:

$$\frac{1}{2\pi i}\ L\frac{f(\tau)d\tau}{\tau-t} = \frac{1}{2\pi i}\int_L \frac{f(\tau)-f(t)}{\tau-t}d\tau + \frac{1}{2}f(t), \tag{3}$$

in which the integral of the right-hand side coincide. The properties of the Cauchy-type integral are used directly to solve two-dimensional problems of the theory of elasticity. For example, the limit values of the singular integral are given in the form [8]:

$$\Phi(z) = \frac{1}{2\pi i}\int_L \frac{\phi(t)\bar{d}t}{t-z}, \tag{4}$$

where $\phi(t)$ belongs to the class of Hölder functions $H^{(\mu)}$.

The joint application of the introductory provisions of the flat problem of the theory of thermoelasticity and the theory of functions of a complex variable or the method of singular integral equations allows for estimating the stress-strain state near a defect of the crack or problematic inclusion type. The asymptotic distributions of the stress tensor and displacement vector caused by introducing the specified defects into the elastic body are significant. For example, in the asymptotic approximation, the combined stress tensor and displacement vector near the vertices of a rectilinear problematic inclusion or crack is given as follows [9]:

$$\begin{gathered} 4G\begin{pmatrix} u \\ v \end{pmatrix} = \frac{K_I^{\pm}}{\rho^*}\sqrt{\frac{r}{2\pi}}\begin{pmatrix} [2(x+\rho^*)+1]\cos\frac{\beta}{2} - \cos\frac{3\beta}{2} \\ [2(x-\rho^*)-1]\sin\frac{\beta}{2} - \sin\frac{3\beta}{2} \end{pmatrix} - \\ -\frac{K_{II}^{\pm}}{\rho^*}\sqrt{\frac{r}{2\pi}}\begin{pmatrix} [2(x+\rho^*)+1]\sin\frac{\beta}{2} + \sin\frac{3\beta}{2} \\ -[2(x-\rho^*)-1]\cos\frac{\beta}{2} - \cos\frac{3\beta}{2} \end{pmatrix} + 0(r^{3/2}), \end{gathered} \tag{5}$$

where $(u, v)$ are the components of the vector of elastic displacements; $K_I^{\pm}$, $K_{II}^{\pm}$ — stress intensity coefficients (coefficients for the singular part of stresses); the signs "$+$" and "$-$" correspond to the right and left vertices of the linear crack-like defect. If $\rho^* = -1$, the given formulas show the known asymptotic distribution near the crack, and if $\rho^* = x$, where $x = 3 - 4\mu$ for plane strain and $x = (3 - \mu)/(1 - \mu)$ for the plane stress state, we obtain the corresponding asymptotics in the case of solid inclusion [9-11].

The stress at the tip of a crack-like defect has a root characteristic of $1/\sqrt{r}$, where $r$ is the distance from the end of the crack or inclusion. At the same time, the coefficients $K_I$ and $K_{II}$ characterize a local increase in the stress level at the top of a crack-like defect, and their values do not depend on the coordinates of this defect. Although the dimension of the stress intensity coefficients, at first glance, seems unusual — $MPa/\sqrt{m}$, these values can be interpreted as a specific stress acting at a distance of $\pi/2$ from the top.

The simulation results make it possible to effectively assess the influence of foreign fillers on the loss of strength of an elastic body containing the specified imperfections. In turn, the exact determination of the order and character of the singularity of stresses near the vertices of an acute-angle imperfection in an elastic material, presented in an analytical form, is required in fracture mechanics for the formulation and recording of the appropriate criterion strength ratios.

It is assumed that the coefficient $G(t)$ belongs to the class $H_0$ and does not become zero anywhere on $C$. Let us agree to understand by $\ln G(t)$ a branch of the logarithmic function that is wholly defined for each arc $a_k b_k$. Obviously, $\ln G(t)$ also belongs to the class $H_0$.

Consider the Cauchy-type integral:

$$\gamma(z) = \frac{1}{2\pi i}\int_\Gamma \frac{\ln G(\tau)d\tau}{\tau - z}. \tag{6}$$

Then the behavior of the function $\exp \gamma(z)$ around the corner $c_j$ is characterized by the formula $\exp \gamma(z) = (z - c_j)^{\alpha_j + \beta_j} \exp \gamma_0(z)$. We will look for the canonical solution to problem (2) in the form:

$$(z) = \exp \gamma(z) \prod_{j=1}^{n} (z - c_j)^{-x_j} = \prod_{j=1}^{n} (z - c_j)^{\alpha_j - x_j + \beta_j} \exp \gamma_0(z), \tag{7}$$

where $x_j$ are integers chosen so that the desired behavior of the function $X(Z)$ in the neighborhoods of the nodes of line $C$ is ensured, namely $-1 < a_j - b_j < 1$.

Remote sensing is one of the effective methods of studying natural phenomena, which is successfully used to study heterogeneous environments, and their characteristics and determine the peculiarities of their evolution [12, 13].

In this case, the impulse response over the entire span does not depend on $t$: $h(\tau, t) = h(\tau)$, which leads to a mathematical model that uses convolution-type equations:

$$\int_{-\infty}^{+\infty} h(t - \tau) s(\tau) d\tau = g(t) \tag{8}$$

or $hS = g$. The simplicity of solving equations of this type (both concerning $s(t)$ and $h(t)$) makes this approach quite effective in many applied problems.

Mathematical models built based on equations of this type make it possible to study processes in heterogeneous environments with variable parameters with the necessary accuracy, which opens up new opportunities for their research.

With their help, we get the boundary condition of the Riemann problem $\Phi^+(x) = G(x)\Phi^-(x) + g(x)$:

$$G(x) = \frac{A(x) - e^{i\pi\lambda}B(x)}{A(x) - e^{-i\pi\lambda}B(x)};$$
$$g(x) = \frac{2i \sin \pi\lambda (\beta - x)^\lambda f(x)}{A(x) - e^{-i\pi\lambda}B(x)}. \tag{9}$$

According to the solution $\Phi(z)$ of the boundary value problem, the solution $\nu(x)$ of the singular integral equation is found from the Abel equation [8]:

$$2i \sin \pi\lambda \int_\alpha^x \nu(\tau) \left(\frac{\beta - x}{x - \tau}\right)^\lambda d\tau = \hat{O}^+(\tilde{0}) - \hat{O}^-(\tilde{0});$$
$$2i \sin \pi\lambda \int_x^\beta \nu(\tau) \left(\frac{\beta - x}{\tau - x}\right)^\lambda d\tau = -e^{-i\pi\lambda}\Phi^+(x) + e^{i\pi\lambda}\Phi^-(x) \tag{10}$$

The behavior at the ends of the segment $[\alpha, \beta]$ of the sought solution $\Phi(z)$ of the Riemann boundary value problem [14] depends on the behavior of the function $\mu(x)$. For example, suppose the solution $\nu(x)$ of the singular integral equation (6) admits integrated singularities at the points $\alpha$ and $\beta$. In that case, the function $\mu(x)$, which corresponds to it, satisfies the interval $(\alpha, \beta)$ condition $H$ with an index greater than $1 - \lambda$ and admits at points $\alpha$ and $\beta$ singularities of orders less than $\lambda$ and 1, respectively. On the other hand, if the function $\mu(x)$ (the difference of the boundary values of the solution of the Riemann boundary value problem) satisfies the condition $H$ with an exponent greater than $1 - \lambda$ on the interval $(\alpha, \beta)$ and allows singularities of orders smaller than $\lambda$ and 1 at points $\alpha$ and $\beta$, then the solution $\nu(x)$ corresponding to them have integrated singularities at points $\alpha$ and $\beta$.

Assume that the coefficients $A(x)$, $B(x)$ of equation (6) satisfy $H$ with an index greater than $1 - \lambda$, the right-hand side of $f(x)$ satisfies the same condition $H$ on the interval $(\alpha, \beta)$ and allows singularities of orders smaller than $\lambda$ at points $\alpha$ and $\beta$. In addition, let the condition hold:

$$\Omega(x) = A_1^2(x) + B_1^2(x) = A^2(x) - 2\cos \pi\lambda A(x)B(x) + B^2(x) \neq 0 \tag{11}$$

Obviously, $|G(x)| = 1$. We denote by $\theta(x) = \arg G(x)$ such a branch of the multivalued function that

$-2\pi < \theta(\alpha) \leq 0$. Then if $\theta(\alpha) < 2\pi$, it is possible to construct an unbounded solution of the Riemann boundary value problem at the end $x = \alpha$; if $\theta(\alpha) \leq 2\pi$, then the solution of the Riemann problem must be bounded at this end. At the end $x = \beta$, the solution of the boundary value problem can be unbounded. The assumption of Hölderian properties of the coefficients on the right side of the equation [15] will ensure that the difference of the boundary values of the solution of the Riemann boundary value problem belongs to class $H$ with an index greater than $1 - \lambda$.

## Research results

The strength and destruction of essential structural elements are significantly influenced by their materials' heterogeneity and defectiveness (structural, technological, and deformation damage). Therefore, when determining the bearing capacity of such elements, there is a need to take into account the micro-heterogeneity and defectiveness of materials and products. Among various defects, cracks, pointed cavities, and extraneous inclusions are of particular importance, as they cause a significant concentration of stresses in structural elements. For example, developing such defects into an unstable trunk crack leads to local or complete destruction. Taking defectiveness into account makes it possible to accurately determine structural elements' bearing capacity accurately.

The intensity of the formation of cracks is determined by the properties of structural components and their orientation in the matrix of metals prone to this type of defect. Therefore, it is advisable to consider the influence of structural parameters on the main criterion of local destruction of $K_{1C}$.

When setting the problem of improving the functional properties of structural elements, the problem arises of assessing the influence of inhomogeneities, the choice of technological parameters, the use of which eliminates the appearance of defects on the treated surfaces. In this regard, it is necessary to create a database for optimizing the thermomechanical state of the surface layer, taking into account its defectiveness, excluding their development to the level of loss of functional properties of structural elements.

Of the available fracture criteria that take into account the local physical and mechanical properties of inhomogeneous materials, the most acceptable for this case are the criteria of the force approach associated with the use of the concept of stress intensity factor (SIF). When loading leads to the stress intensity $K_I$ becoming equal to the limiting value $K_{Ic}$, the crack-like defect turns into the main crack.

The effect of the initial piecewise homogeneity of processed materials (parts with coatings) on thermomechanical processes is carried out by discontinuous solutions. By them, we mean such solutions that satisfy the Fourier thermal conductivity and Lame elasticity equations everywhere except for the boundaries of defects. When crossing the boundary, the displacement and stress fields suffer discontinuities of the first kind, i.e., their jumps appear $\langle u \rangle$, $\langle v \rangle$, $\langle \sigma_x \rangle$, $\langle \tau_{xy} \rangle$.

The stress-strain state of a layered half-plane is also estimated by the method of discontinuous solutions. The interfaces $x = a_k (k = 0)$ are considered defects when passing through which the displacement and stress fields suffer discontinuities.

Of most significant interest is the behavior of stresses in the vicinity of the vertices of defects such as cracks, pointed inclusions, and structural imperfections, i.e., stress features at $y \to \pm l_k$. The nature of the stress field near the end of the defect, obtained in the framework of the classical theory of elasticity, is determined by the stress intensity factors $K_I = iK_{II}$.

Thus, the study of the stress intensity at the vertices of a defect with a length of $2l$, located at a depth of $\sigma^*$, when the heat flux $q$ is set on the surface of the body $(x = 0, |y| \leq a^*)$, made it possible to establish the limiting value this flow $q^*$ at which the specified defect begins to develop into the main crack:

$$q^* = \frac{2\sqrt{3}\lambda(1-\nu)K_{4C}}{\alpha_2 El\sqrt{\pi l}\sigma^*} \tag{12}$$

For defect-free machining of steels and alloys with crack-like defects and inclusions, when choosing machining modes and tool characteristics, one should be guided by the limiting values of the heat flux formed during machining so that hereditary defects do not leave the equilibrium state.

The surface layer of processed materials contains heterogeneities and imperfections of hereditary origin, which have varying degrees of randomness. Therefore, a probabilistic-statistical approach is necessary to study the causes of the appearance of defects, in addition to the deterministic one.

To elucidate the role of the heat stress of micro-cutting in the mechanism of crack initiation, taking into account the heterogeneity of the material being processed, the mechanical and thermal characteristics of the impulse components of machining were determined. It has been established that the heat flux from them is a variable value determined by the formula $q = c\sqrt{\tau}$, where $c$ is a coefficient that takes into account the thermophysical properties of the metal and processing modes, $\tau$ is the contact time of the pulsed component of the tool with the machined surface of the experimental studies confirm the established pattern of changes in the heat flux density from the impulse component.

A comparison of the probabilities of micro destruction from the thermal stress of the impulse components showed that the highest probability is determined by thermoelastic stresses, the magnitude of which is associated with the intensity of the heat flux.

The influence of the design parameters of the tool on the thermomechanical state of the surface layer was determined using the model problem (7), (10), and the boundary conditions in the form:

$$q(y,\tau) = \frac{c\sqrt{\tau}}{\lambda}[H(y) - H(y2a^*)] \sum_{k=0}^{n} \sigma(y + kl - v_{kp}\tau) \tag{13}$$

where $H(y)$ is the Heaviside function; $\sigma(y)$ - Dirac delta function; $n$ is the number of grains passing in the contact zone during the time ; $\lambda$ - thermal conductivity of the product material; $c\sqrt{\tau}$ is the heat flow from a single grain-pulse component of the tool; $v_g$, $v_{kp}$, $t_{gr}$ processing modes, $2a^*$ is the length of the arc of contact between the tool and the workpiece; $l^*$ is the distance between impulse components. The maximum values of the instantaneous temperature $T_M$ were obtained theoretically and experimentally confirmed, from the pulse component to the constant component -$T_K$, which are used further as criteria values in determining the conditions for the formation of defects such as burns and their depth.

The influence of processing modes on the qualitative characteristics of the surface layer was studied depending on the dominant factors. The adequacy of the theoretically established dependences of the contact temperature on the processing parameters and the physical and mechanical properties of the processed material was checked. The controlled values were: microhardness, residual stresses, tensile strength, and the presence of microcracks on the treated surface. The structural stress state of the surface layer of the material, which is formed during its processing, affects the fracture toughness $K_{Ic}$. Of the technological parameters, the influence of the depth of processing as the dominant factor in the occurrence of thermomechanical phenomena on the value of the crack resistance coefficient was studied. The decrease in crack resistance is associated with an increase in tensile residual stresses.

The influence of technological heredity on the crack resistance of metals during grinding was studied using the fracture mechanics parameter $K_{Ic}$, which considers the stress-strain state's dependence on the surface layer's structural components.

The study of the role of the heterogeneity of the coating structure in the mechanism of crack resistance reduction was carried out using the theoretically established criterion of local failure in the form of the following inequality:

$$l_0 < \frac{Da\lambda^2 v_q^u K_c^2}{\pi^2 \left[ cv_{kp} G(1+\nu) a_t \left( 1 - 2xp\left( \frac{v_q\sqrt{Dt}}{a\tau} \right) \right) \right]} \tag{14}$$

where $\upsilon_{kp}$, $\upsilon_q$, $t$ are grinding modes; $D$, $C$ - tool parameters; $\lambda$, $a$ - thermophysical characteristics of the treated coating; $K_C$ - crack resistance of this coating; $G$ is the modulus of elasticity; $\nu$ - Poisson's ratio; $a_t$ - temperature coefficient of linear expansion; $l$ is the characteristic linear dimension of the structural parameter (structural defect).

The development of technological criteria for controlling the process of defect-free grinding was based on the established functional relationships between the physical and mechanical properties of the processed materials and the main technological parameters.

The quality of the machined surfaces will be ensured if, using the controlling of the technological parameters, such as processing modes, lubricating-cooling media, and tool characteristics are selected that the current values of the grinding temperature $T(x, y, \tau)$ and the heat flux $q(y, \tau)$, stress $\sigma(M)$ and grinding forces $P_y$, $P_z$ coefficient $K_{Ic}$ will exceed their limit values.

Implementation of the system of limiting inequalities in terms of the values of the temperature itself and the depth of its propagation in the form:

$$T(x,y,\tau) = \frac{C}{2\pi\lambda} \sum_{k=0}^{n} H\left(\tau - \frac{kl}{v_{kp}}\right) H\left(\frac{L+kl}{v_{kp}}\right) \times \\ \times \int_{\gamma_1}^{\gamma_2} f(x,y,\tau,\tau')d\tau' \leq [T]_M \tag{15}$$

$$T([h],0,\tau) = \frac{C}{2\pi\lambda} \sum_{k=0}^{n} H\left(\tau - \frac{kl}{v_{kp}}\right) H\left(\frac{L+kl}{v_{kp}}\right) \times \\ \times \int_{\gamma_1}^{\gamma_2} \psi(x,\tau,\tau')d\tau' \leq [T]_{\text{с.п.}} \tag{16}$$

$$T_k(0,y,\tau)=\frac{Cv_{kp}}{\pi\lambda l\sqrt{v_g}}\int_a^\tau\int_{-e}^{e}\frac{\chi(\eta,t)e^{\frac{(y-\eta)^2}{4(\tau-t)}}}{2\sqrt{\pi(\tau-t)}}\times$$
$$\left\{\frac{1}{\sqrt{\pi(\tau-1)}}+\gamma e^{\gamma^2(\tau-t)}\left[1+\Phi\left(\gamma\sqrt{\tau-t}\right)\right]\right\}d\eta dt\le[T] \quad (17)$$

$$T_k^{max}(L,0)\frac{Cv_{kp}\alpha}{\lambda l v_q^2}\sqrt{\frac{\alpha}{\pi}}\left[1-exp\left(-\frac{v_q\sqrt{Dt_{шл}}}{\alpha}\right)\right][T] \quad (18)$$

allows avoiding the formation of grinding burns and can serve as the basis for designing grinding cycles according to the thermal criterion.

The processing of materials and alloys without grinding cracks can be ensured if the stresses formed in the zone of intensive cooling are limited to the limit values:

$$\sigma(x,\tau)\frac{1+\nu}{1-\nu}_{t_k}\left(\frac{x}{2\sqrt{\alpha\tau}}\right)[\sigma_{лч}]_{max} \quad (19)$$

In the case of the dominant influence of hereditary heterogeneity on the intensity of the formation of grinding cracks, it is necessary to use criteria that include deterministic relationships between technological parameters and the properties of the heterogeneities themselves. As such, you can use the limitation of the stress intensity factor:

$$K=\frac{1}{\pi\sqrt{l}}\int_{-e}^{e}\sqrt{\frac{l+t}{l-t}}\{\sigma_x,\sigma_y\}d\,t\le K_{Ic} \quad (20)$$

or providing, with the help of control technological parameters, the limiting value of the heat flux, at which the balance of structural defects is maintained:

$$q^*=\frac{P_z v_{kp}\alpha_S}{\sqrt{Dt_{шл}}}\le\frac{\sqrt{3}\lambda K_{Ic}}{Hl\sqrt{\pi l}\sigma} \quad (21)$$

Defect-free grinding conditions can be implemented using information about the structure of the material being processed. So, in the case of the general nature of structural imperfections with a length of $2l$, their regular location relative to the contact zone of the tool with the workpiece, it is possible to use the defect equilibrium condition as a criterion relation in the form:

$$l_0<\frac{K_C^2}{x[GT_k(1+\nu)\alpha_t]^C} \quad (22)$$

In this formula, the technological part is connected with the value of the contact temperature $T_k$ with the grinding conditions.

The above inequalities link the longitudinal characteristics of the temperature and force fields with the control and technological parameters. They set the range of combinations of these parameters that satisfy the obtained thermomechanical criteria. At the same time, the processed material's properties are considered, and the required quality of products is guaranteed.

The fundamental principles of devices for automatic stabilization of the grinding temperature and the quality characteristics of the workpieces have been developed. Unlike known devices, the proposed circuits of such devices should contain blocks for determining the current values of the burn depth of the polished surface, contact and instantaneous temperatures, thermal exposure time, and stress intensity. From the output of these blocks, the values of the indicated values are fed into the control parameter selection logic block. Finally, the power amplifiers send the corresponding commands to the machine's executive bodies, where the limit values of the controlled values are worked out (Fig. 2).

Stochastic micro-heterogeneity and defects are formed in structural elements during manufacture and operation. The surface layer of products contains inhomogeneities and defects of hereditary origin, which are associated with one degree of randomness. Therefore, it is necessary to use a probabilistic-statistical approach when investigating the causes of hereditary and deterministic cracks in structural elements [16].

The stochastic model of the formation of cracks during the grinding of metals of heterogeneous structure is built based on a complex approach based on the results of the deterministic theory of the development of individual defects and methods of probability theory. The surface layer is considered an environment weakened by random defects - cracks, and inclusions, which determine that they do not interact with each other, the parameters of which are random values with known probability distribution laws. The probability of destruction of the surface layer is studied depending on different types of probability distributions of defects' sizes (length, depth) and their orientation. Probabilistic characteristics of the limiting heat flow are considered from the same positions. It was established that an increase in the homogeneity of the material leads to an increase in the heat flow value, which corresponds to a fixed probability of destruction. Therefore, when investigating the limited state of structural elements weakened by defects, and building on this basis a well-founded theory of their strength and destruction, in addition to the deterministic one, it is necessary to use a probabilistic-statistical approach [16].

A structural element is considered an environment weakened by random defects (cracks, defining inclusions, the parameters of which are random values with known laws of their probability distribution).

For an element containing $n$-random cracks, the probability of failure is calculated assuming the weakest link, the probability distribution of which is static.

## Conclusions

The limiting value of the heat flux for the balanced state of the crack, which has the length of the weakest link under the thermal influence on the structural elements, which have uniformly scattered, randomly distributed defects of the crack type, is determined.

The developed model takes into account the influence of inhomogeneities of technological origin (starting with the workpiece and ending with the finished product), which arise in the surface layer during the manufacture of structural elements, on its destruction.

Determining the stress intensity around the peaks of crack-type defects and comparing it with the crack resistance criterion for the material of the structural element allows the solution of the singular integral equation with the Cauchy kernel. Modeling of thermomechanical processes makes it possible to obtain a criterion ratio of the conditions of the balanced state of the defect depending on the contact temperature gradients.

The mechanism of formation and development of defects, such as cracks in functional-gradient materials of the heterogeneous structure under the influence of thermomechanical phenomena accompanying the manufacturing and operation of structural elements, was studied. The basic principles of the device for automatic stabilization of the grinding temperature and quality characteristics of the processed parts have been developed.

The proposed schemes, in contrast to known devices, should contain blocks — determination of the current values of the depth of ignition of the surface being polished, determination of the contact and instantaneous temperatures, the time of thermal exposure, and the intensity of stresses.

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# КОНТРОЛЬ ТА УПРАВЛІННЯ ТЕРМОМЕХАНІЧНИМИ ЯВИЩАМИ ПРИ МЕХАНІЧНІЙ ОБРОБЦІ ВИРОБІВ ІЗ МАТЕРІАЛІВ НЕОДНОРІДНОЇ СТРУКТУРИ

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*У статті представлено інформаційну технологію аналізу та синтезу моделей забезпечення якісних характеристик робочих поверхонь виробів при механічній обробці з урахуванням дефектів структури матеріалу. Описано різноманітні мікродефекти, розвиток яких під впливом механічної обробки призводить до появи тріщин та їх збільшення і, як наслідок, локального або повного руйнування. Розв'язано задачу термопружності для тіл, ослаблених неоднорідностями. Розглянуто ймовірнісно-статистичний підхід у підвищенні точності ідентифікації технологічних процесів механічної обробки, розробці та впровадженні нових, більш ефективних методів і засобів інформаційного та комп'ютерного моделювання. Розроблена модель враховує вплив неоднорідностей технологічного походження (починаючи від заготовки й закінчуючи готовим виробом), які виникають у поверхневому шарі* *при виготовленні елементів конструкції, аналіз яких дозволяє створити інформаційну базу критеріїв, реалізація яких дозволяє запобігти втраті функціональних властивостей відповідальних елементів. Удосконалено математичні моделі динаміки термомеханічних процесів, що супроводжують механічну обробку виробів із матеріалів неоднорідної структури, у вигляді просторово-нестаціонарної постановки задачі на основі систем диференціальних рівнянь термопружності в часткових похідних та умовах розриву. на дефекти пружно-деформаційних характеристик обробленого матеріалу, що на відміну від існуючих дозволило підвищити точність ідентифікації математичних моделей загалом. Розроблено математичні моделі системи оцінювання ефективності функціонування технологічних комплексів механічної обробки, які дають змогу визначити зв'язок між параметрами стану оброблених поверхонь та основними керуючими технологічними характеристиками, що забезпечують необхідні властивості функціонально градієнтні матеріали. Нарешті, результати моделювання дають можливість створити ефективну інформаційну технологію, яка дає змогу суттєво зменшити втрати функціональних властивостей гетерогенних систем.*

**Ключові слова:** конструктивний елемент, поверхневий шар, тепловий потік, оброблюваний матеріал, контрольована величина.

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